Optimal Structural Design with Control Gain Norm Constraint

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A structure/control system optimization problem has been formulated with constraints on the closed-loop eigenvalue distribution, structural frequencies, and the minimum Frobenius norm of the required control gains. Suggested is simultaneous optimization where, at each iteration, the control objective function is minimized first with the closed-loop eigenvalue constraints and then structural optimization is performed to satisfy the constraints on the optimal control gain norm and structural frequencies. The feasibility of the approach is demonstrated on a two-bar truss structure. For each locally optimal design, response simulations have been made and control efforts observed. Qualitative aspects of the optimal designs are also included and general conclusions drawn.

Introduction

A GREAT deal of research[1] is currently in progress on designing active vibration control systems for large flexible space structures to reduce the structural response resulting from some initial disturbance to acceptable levels within a reasonable time span. In addition, it is important that this objective be achieved in some optimal manner, i.e., least total weight for the structural design and least control cost for the active control design. Recently, there has been considerable interest in the simultaneous integrated design of the structure and vibration control system to produce a truly optimum configuration[2-8] that results in performance as well as cost improvements.

In our earlier work,[4] a structural/control optimization problem was formulated that could directly affect the required control effort and the transient behavior of the control system. In Ref. 4, for a chosen structural objective function, explicit eigenvalue constraints on the closed-loop control system were imposed and the Frobenius norm of the required optimum control gains was introduced as an inequality constraint to be satisfied by the structural optimization procedure. Thus, an optimum control problem was solved within the structural optimization problem. The results of Ref. 4 gave multiple optima and additional constraints were suggested to better understand the nature of the structure control optimization problem. Essentially, the Frobenius norm of the control gains represents an expected valued of a quadratic control effort. Thus, by putting a constraint on the expected value one hopes to monitor the required control effort.

In this paper, we improve the solution of the simultaneous structure/control design problem posed in Ref. 4 by including additional constraints. In addition to the equality constraints on the closed-loop eigenvalues, inequality and equality constraints on the structural natural frequencies are considered in this paper. Furthermore, in some examples, an upper bound inequality constraint on the control gain norm is used. In contrast, only equality constraints on the control gain norm were used in Ref. 4. It is hoped that by introducing new constraints to the optimization problem, the important features and trends of the structure/control optimization will be identified. The ultimate objective is to reduce number of multiple local minima encountered in Ref. 4.

A brief formulation of the optimization problem is given followed by the description of the optimization procedure. A number of optimal solutions are given for a flexible two-bar truss structure. A discussion of results is presented by pointing out certain features of the optimal solutions.

Problem Formulation

Consider a structural dynamic system described by

$$M\ddot{q} + Kq = DF \tag{1}$$

where $q(t)$ and $F(t)$ represent n and ℓ component vectors of displacements and inputs, respectively; M and K are $n \times n$ positive definite mass and stiffness matrices; and D is a $n \times m$ input distribution matrix. For control design purposes, the corresponding state-space dynamics is

$$\dot{x} = Ax + BF \tag{2}$$

where

$$x = [q^T \quad \dot{q}^T]^T \tag{3}$$

Presented as Paper 87-0019 at the AIAA 25th Aerospace Sciences Meeting, Reno, NV, Jan. 12-15, 1987; received Feb. 17, 1987; revision received Sept. 10, 1987.

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$$A = \left[\begin{array}{c|c} 0 & I \\ \hline -M^{-1}K & 0 \end{array}\right] \tag{4}$$

$$B = \left[\begin{array}{c} 0 \\ \hline M^{-1}D \end{array}\right] \tag{5}$$

The control input is a linear state feedback law of the form

$$F = -Gx \tag{6}$$

The gain matrix G is computed such that the closed-loop dynamics

$$\dot{x} = (A - BG)x = A_{CL}x \tag{7}$$

satisfies the $2n$ eigenvalue constraints

$$\rho_i \le \bar{\rho}_i, \qquad i = 1,2,\ldots,2n \tag{8}$$

where ρ_i and $\bar{\rho}_i$ correspond to the ith closed-loop eigenvalue and the constraint value, respectively.

In Eq. (8), an equality constraint implies an eigenvalue allocation. On the other hand, inequality constraints on the real parts of eigenvalues represent stability margin requirements. Let the consecutive pairs of eigenvalues be written as

$$\rho_{2i-1,2i} = \alpha_i \pm j\beta_i, \qquad i = 1,2,3,\ldots,n \tag{9}$$

where $j = \sqrt{-1}$. The $2n$ eigenvalue constraints can then be written as

$$\alpha_j - \bar{\alpha}_j \le 0, \qquad j = 1,2,\ldots,n \tag{10}$$

and

$$\beta_j - \bar{\beta}_j \le 0, \qquad j = 1,2,\ldots,n \tag{11}$$

which represent constraints on the real and imaginary parts of the closed-loop eigenvalues where $\bar{\alpha}_j$ and $\bar{\beta}_j$ are specified values, $\alpha_j < 0$ and $\beta_j > 0$. The closed-loop damping ratios are given by

$$\xi_j = \frac{-\alpha_j}{\left(\alpha_j^2 + \beta_j^2\right)^{\frac{1}{2}}} \tag{12}$$

or one may consider the closed-loop damping ratio constraints

$$\xi_j - \bar{\xi}_j \ge 0 \tag{13}$$

which will insure minimum damping ratios of $\bar{\xi}_j$.

Associated with the closed-loop eigenvalue constraints, we consider the quadratic control measure

$$J = \int F^T Q^T Q F \, dt = \int x^T G^T R G x \, dt \tag{14}$$

where Q is a symmetric positive definite matrix. The expectation and trace operators by E and Tr, respectively we can define an expected quadratic control measure S in the form

$$S = E(J) = E\int_0^T x^T G^T R G x \, dt = TrG^T R G x \tag{15}$$

where $X = E\{\int_0^T x x^T \, dt\} = \text{const}$. Because the control system is assumed to be stable, the integral will be finite and the X represents an integral covariance matrix over the ensemble of possible state trajectories. Without loss of generality, we can take

$$X = E\int_0^T \{x x^T\} \, dt = I$$

where I is $2n \times 2n$ identity matrix and the off-diagonal terms are uncorrelated. In this case, the expected control cost becomes

$$S = TrG^T R G \tag{16}$$

Alternatively, one may consider the control effort with unit weighting $Q = 1$

$$S = E\left\{\int F^T F \, dt\right\} = E\int_0^T TrFF^T \, dt$$
$$= TrGE\int_0^T \{x x^T\} \, dt \, G^T = TrG^T X G \tag{17}$$

Hence, upon comparing Eqs. (16) and (17) over the weighting matrix R can also be interpreted as the integral covariance matrix X of the state vector, $X = R$. As a control objective, we shall seek to minimize the expected control measure S and note that it is not the weighed Frobenius norm of the control gains,

$$S_G = TrG^T R G > 0 \tag{18}$$

It follows that the expected control measure S can be minimized by minimizing S_G, the Frobenius norm of the control gains.

The norm S_G of a feasible control gain matrix G is an explicit function of control gain elements g_{rs} ($r = 1,2,\ldots,l$; $s = 1,2,\ldots,2n$), which are in turn implicit functions of structural parameters and closed-loop eigenvalues ρ_i. In general, a natural frequency ω_r of an uncontrolled structural system is an embodiment of structural parameters. The moduli $|\rho_i|$ of the closed-loop eigenvalues ρ_i represent the natural frequencies of the controlled structural system. The magnitudes of the control gain parameters are strongly related to the separations between the structural natural frequencies ω_r and the controlled natural frequencies ρ_i. Larger shifts in the eigenvalues require larger control gains.[9] From this perspective, along with closed-loop eigenvalues constraints, we can also impose constraints on the open-loop eigenvalues, that is, on the structural frequencies ω_r to monitor the amount of control gains. This is consistent with the objective of minimizing S_G.

Next, for structural design purposes, we introduce the structural objective function (weight or mass)

$$W = W(p) \tag{19}$$

where p is a structural parameters vector of dimension m_s with the parameter constraints $p_k > \bar{p}_k$ $k = 1,2,\ldots,m_s$, where $\bar{p}_k$ denotes the minimum allowable values of the parameters.

Hence, we pose the structure/control system optimization problem,

$$\text{Minimize } W(p) \tag{20}$$

subject to

$$\text{Minimum } S_G \le \bar{S}_G \tag{21}$$

$$\alpha_j - \bar{\alpha}_j \le 0, \qquad j = 1,2,\ldots,n \tag{22}$$

$$\beta_j - \bar{\beta}_j \le 0, \qquad j = 1,2,\ldots,n \tag{23}$$

$$\omega_j - \omega_j \le 0, \qquad j = 1,2,\ldots,c \tag{24}$$

where an overbar denotes specified values. Equations (20–24) describe a nested optimization problem constituting a simultaneous structure/control design.

Optimization Procedure

The simultaneous optimization process can be considered as follows:

1) Corresponding to any set of structural parameters P first minimize the control objective function S_G subject to the closed-loop eigenvalue constraints to obtain the optimal gains.

2) With the optimal gains available, optimize the structural objective function $W(p)$ subject to the constraints on the structural frequencies and the constraint on the minimum value of the control gain S_G.

The solution of the optimization problem requires the sensitivities of the objective functions and the constraint functions to the design parameters p_k and g_{rs}. The sensitivity of the gain norm S_G and the eigenvalue ρ_j are given by

$$\frac{\partial S_G}{\partial g_{rs}} = \sum_{j=1}^{l} 2R_{rj}g_{js} \tag{25a}$$

$$\frac{\partial \rho_j}{\partial g_{rs}} = t_j^{*T} b_k t_{lj} \tag{25b}$$

$$j = 1,2,\ldots,2n, \qquad r = 1,2,\ldots,l \tag{26}$$

where t_j^* and t_j are the jth left and right eigenvectors of A_{CL}, b_k is the kth column of B, and t_{lj} the lth element of t_j.

The sensitivity of the objective function W, gain norm S_G, and structural frequencies ω_r, with respect to the structural design variables, are given by

$$\frac{\partial W}{\partial p} = W_p \tag{27}$$

$$\frac{\partial S_G}{\partial_{pk}} = \sum_{s}^{2n}\sum_{r}^{l}\sum_{m}^{l} R_{rm}\left(\frac{\partial g_{rs}}{\partial p_k}g_{ms} + g_{rs}\frac{\partial g_{ms}}{\partial p_k}\right) \tag{28}$$

$$\pm j\frac{\partial \omega_r}{\partial p_k} = e_i^{*T}\frac{\partial A}{\partial p_k}e_i$$

$$i = 1,2,\ldots,2n; \qquad r = 1,2,\ldots,n; \qquad j = \sqrt{-1} \tag{29}$$

where e_i^* and e_i are the left and right eigenvectors of A. We note that the sensitivities of the control gains with respect to structural design variables are also required in Eq. (28).

Illustrative Example

The concepts discussed above were applied to a two-bar truss structures shown in Fig. 1[2-4] for which closed-form formulas for the required sensitivities could be obtained. For the geometry shown, the equations of motion for the finite element model of the truss are

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{Bmatrix} \ddot{U}_1 \\ \ddot{U}_2 \end{Bmatrix} + k\begin{bmatrix} (A_1 + A_2) & 2(A_1 - A_2) \\ 2(A_1 - A_2) & 4(A_1 + A_2) \end{bmatrix}\begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} \cos\theta \\ \sin\theta \end{Bmatrix} F(t) \tag{30}$$

in which A_1 and A_2 are the cross-sectional areas of the bars and $k = E/(5L)$ a stiffness parameter, E the elastic modulus of the bars, and L the length of members. The control force $F(t)$ is located at the vertex of the truss with θ showing its line of action with respect to the horizontal. U_1 and U_2 are the horizontal and vertical displacements of the vertex. Furthermore, a nonstructural mass of two units was placed at the vertex and the structural mass was ignored for simplicity. The control law was the form

$$F = -[g_1 \quad g_2 \quad g_3 \quad g_4][U_1 \quad U_2 \quad \dot{U}_1 \quad \dot{U}_2]^T = -Gx \tag{31}$$

The structural and control objective functions were taken as

$$W = \sum_{k=1}^{2} \gamma A_k L \tag{32}$$

$$S_G = Tr, G^T G = \sum_{i=1}^{4} g_i^2 \tag{33}$$

with the specified equality constraints on the closed-loop eigenvalues as

$$\bar{\alpha}_1 = -0.0228, \qquad \bar{\alpha}_2 = -0.361 \tag{34}$$

$$\bar{\beta}_1 = 1.17, \qquad \bar{\beta}_2 = 4.81 \tag{35}$$

and on the gain norm

$$\min S_G = \bar{S}_G \quad \text{or min} \quad S_G \le \bar{S}_G$$

where

$$\bar{S}_G = 1500 \tag{36}$$

In addition, the following constraints were imposed on the magnitudes of open-loop eigenvalues that are the structural natural frequencies

$$\omega_1 = \bar{\omega}_1 = 1 \text{ rad/s} \tag{37}$$

and

$$\omega_2 \ge \bar{\omega}_2 \qquad \text{or} \qquad \omega_2 = \text{free} \tag{38}$$

In Eq. (32), γ denotes the specific weight. The constraint values were selected arbitrarily. The minimum allowable values of structural cross sections were taken to be 10 units. The other parameters[2-4] were selected as $\theta = 45$ deg, $\gamma = 0.001$, $E = 1$. The design parameters are the two cross-sectional areas A_1 and A_2 and the four control gain parameters g_i ($i = 1,\ldots,4$).

For the solution of this single-input problem, optimum control gains are unique (hence, a globally optimal) and a closed-form expression for them can be obtained at each design iteration corresponding to the specified closed-loop eigenvalues and structural parameters. However, we note that for multi-input designs, the optimal control gains are not unique (locally optimal) and closed-form expressions cannot be obtained. Following the control optimization, the design variables A_1 and A_2, associated with the optimal structural

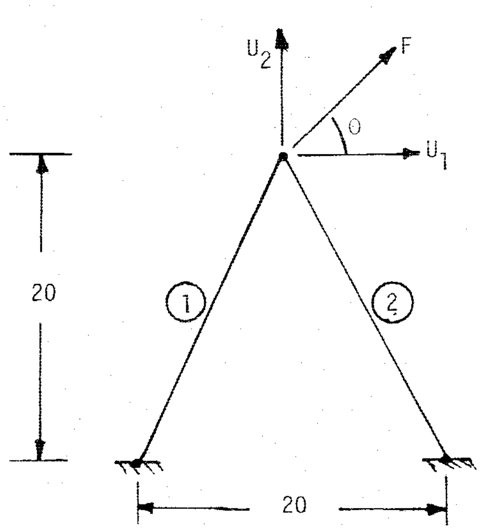

Fig. 1 Two-bar truss.

objective function W, subject to the constraints on the gain norm S_G and natural frequencies ω_1 and ω_2, were obtained numerically by NEWSUMT-A.[10] This program is based on an extended interior penalty method of constrained optimization and modified Newton's method of unconstrained minimization.

Numerical Results

A two-bar truss was optimized with equality and inequality constraints on the control gain norm and the structural frequencies. For case I, the structural weight was minimized with equality constraints on the control gain norm $S_G = 1500$ and the fundamental structural frequency $\omega_1 = 1.0$. ω_2 was free. Different combinations of the initial design variables were used. Depending on the initial design, four different optimum designs were obtained, as shown in Table 1. For initial designs I1–I3, optimum design D1 was obtained with the structural weight $W = 6.417$ and $\omega_2 = 2.327$. Design D2 was obtained with weight $W = 17.095$ and $\omega_2 = 4.012$ with initial designs I4 and I5. With I16–I18 as the initial designs, the optimum design D3 was obtained, which weighed 30.952 with $\omega_2 = 4.012$. Finally, initial design I9 gave D4 as the optimum design with $W = 42.687$ and $\omega_2 = 6.457$. It was found that there was no optimum design with $\omega_2 > 6.457$. The main difference between these designs was the magnitude of the second structural frequency ω_2, which caused the changes in the optimum weights. Each of these optimum designs belongs to a different feasible design of the design space. More than one local minimum exist, indicating that case I was underconstrained. Therefore, additional constraints were imposed in a subsequent trial in order to obtain a possibly unique optimum solution irrespective of the starting point. For case II, a constraint on the second structural frequency $\omega_2 \geq 2$ was imposed in addition to other constraints. This reproduced the results given in Table 1, depending on the initial design, since the second structural frequency ω_2 was greater than 2.0 for all optimum designs D1–D4. For subsequent cases, the constraint on the control gain norm was changed to an inequality constraint $S_G \leq 1500$, with an equality constraint on the fundamental frequency $\omega_1 = 1.0$ and different lower bounds were specified for the second structural frequency ω_2. For case

Table 1 Two-bar truss design, $S_G = 1500$, $\omega_1 = 1.0$, ω_2 free (case I) or $\geqslant 2$ (case II)

Design		A_1	A_2	W	$-g_1$	$-g_2$	$-g_3$	$-g_4$	S_G	ω_1	ω_2
I1	init	100.	100.	4.472	−11.17	−45.95	−1.44	−0.73	2239.2	0.946	1.892
I2	init	500.	500.	22.361	12.33	−18.85	0.29	−2.46	513.5	2.115	4.23
I3	init	700.	700.	31.305	12.56	6.21	0.42	−2.59	203.1	2.502	5.004
D1	opt	203.98	82.98	6.417	−16.73	−34.89	−1.39	−0.78	1500.	1.0	2.327
I4	init	800.	800.	35.777	12.31	19.11	0.45	−2.62	523.7	2.675	5.35
I5	init	100.	500.	13.416	19.18	−51.0	0.97	−3.14	2980.4	1.150	3.478
D2	opt	72.70	691.83	17.095	14.36	−35.77	1.33	−3.50	1500.0	1.0	4.012
I6	init	900.	900.	40.24	11.93	32.13	0.48	−2.66	1182.2	2.837	5.674
I7	init	1000.	1000.	44.721	11.47	45.25	0.51	−2.68	2186.5	2.991	5.981
I8	init	500.	1000.	33.541	4.07	21.02	0.92	−3.09	469.0	2.40	5.271
D3	opt	71.29	1312.9	30.952	−16.84	34.62	1.71	−3.88	1500.0	1.0	5.473
I9	init	1000.	500.	33.541	15.22	9.87	−0.05	−2.12	333.7	2.40	5.27
D4	opt	1838.1	70.9	42.687	15.55	35.41	−0.84	−1.33	1500.0	1.00	6.457

Table 2 Two-bar truss design, $S_G \leqslant 1500$, $\omega_1 = 1.0$, $\omega_2 \geqslant 2$ (case III)

Design		A_1	A_2	W	$-g_1$	$-g_2$	$-g_3$	$-g_4$	S_G	ω_1	ω_2
I1	init	100.	100.	4.472	−11.17	−45.95	−1.44	−0.73	2239.2	0.946	1.892
I2	init	500.	500.	22.361	12.33	−18.85	0.29	−2.46	513.5	2.115	4.23
I4	init	800.	800.	35.777	12.31	19.11	0.45	−2.62	523.7	2.675	5.35
I9	init	1000.	500.	33.541	15.22	9.87	−0.05	−2.12	333.7	2.4	5.271
D1	opt	203.98	82.98	6.417	−16.73	−34.89	−1.39	−0.78	1500.	1.0	2.327
I8	init	500.	1000.	33.541	4.07	21.02	0.92	−3.09	469.0	2.4	5.271
I7	init	1000.	1000.	44.721	11.47	45.25	0.51	−2.68	2186.5	2.991	5.981
I10	init	1500.	1500.	67.082	8.37	111.6	0.58	−2.75	12531.	3.663	7.326
I11	init	2000.	2000.	89.443	4.71	178.5	0.62	−2.78	31891.	4.23	8.459
D2	opt	72.70	691.	17.095	14.36	−35.77	1.33	−3.50	1500.0	1.0	4.012

Table 3 Two-bar truss design, $S_G \leqslant 1500$, $\omega_1 = 1.0$, $\omega_2 \geqslant 3.0$ (case IV)

Design		A_1	A_2	W	$-g_1$	$-g_3$	$-g_4$	S_G	ω_1	ω_2	
I1	init	100.	100.	4.472	−11.17	−45.95	−1.44	−0.73	2239.2	0.946	1.892
I2	init	500.	500.	22.361	12.33	−18.85	0.29	−2.46	513.5	2.115	4.23
I4	init	800.	800.	35.777	12.31	19.11	0.45	−2.62	523.7	2.675	5.35
D5	opt	371.53	75.70	10.0	−15.24	−26.24	−1.20	−0.97	923.5	1.0	3.00
I8	init	500.	1000.	33.541	4.07	21.02	0.92	−3.09	469.0	2.4	5.271
I7	init	1000.	1000.	44.721	11.47	45.25	0.51	−2.68	2186.5	2.991	5.981
I11	init	2000.	2000.	89.443	4.71	178.5	0.62	−2.78	31891.	4.23	8.459
D2	opt	72.70	691.	17.095	14.36	−35.77	1.33	−3.50	1500.0	1.0	4.012

Table 4 Two-bar truss design, $S_G \leqslant 1500$, $\omega_1 = 1.0$, $\omega_2 \geqslant 4.0$ (case V)

Design		A_1	A_2	W	$-g_1$	$-g_2$	$-g_3$	$-g_4$	S_G	ω_1	ω_2
I1	init	100.	100.	4.472	−11.17	−45.95	−1.44	−0.73	2239.2	0.946	1.892
I4	init	800.	800.	35.777	12.31	−19.11	0.45	−2.62	523.7	2.675	5.35
I7	init	1000.	1000.	44.721	11.47	45.25	0.51	−2.68	2186.5	2.991	5.981
I8	init	500.	1000.	33.541	4.07	21.02	0.92	−3.09	469.0	2.4	5.271
I10	init	1500.	1500.	67.082	8.37	111.6	0.58	−2.75	12531.	3.663	7.326
I11	init	2000.	2000.	89.443	4.71	1781.5	0.62	−2.78	31891.	4.23	8.459
I12	init	5000.	5000.	223.6	−19.6	582.3	0.68	−2.85	339420.	6.687	13.375
D2	opt	72.7	691.83	17.096	14.36	35.77	1.33	−3.50	1500.	1.0	4.012
I2	init	500.	500.	22.361	12.33	−18.85	0.292	−2.46	513.5	2.115	4.23
I9	init	1000.	500.	33.541	15.22	9.87	−0.05	−2.12	333.7	2.4	5.271
D6	opt	687.57	72.72	17.000	−8.86	−12.83	−1.02	−1.16	245.4	1.0	4.0

Table 5 Properties of optimum designs[a]

Optimal design	ω_1	ω_2	λ_1	λ_2	S_G	J	J/W
D1	1.0	2.327	1.17022	2.073	1500	1834	286.1
D2	1.0	4.012	1.17022	1.202	1500	333	19.4
D3	1.0	5.473	1.17022	0.881	1500	394	12.7
D4	1.0	6.457	1.17022	0.747	1500	2223	52.0
D5	1.0	3.0	1.17022	1.608	923	1119	111.9
D6	1.0	4.0	1.17022	1.206	245	341	20.0

[a] $|\rho_1| = 1.17022$, $|\rho_2| = 4.82353$, $\zeta_1 = 0.0195$, $\zeta_2 = 0.075$.

III $\omega_2 \geq 2$, for case IV $\omega_2 \geq 3$, and for case V $\omega_2 \geq 4$. The results for these cases are given in Tables 2–4, respectively. Eight different initial designs were tried for case III. This investigation resulted in the two optimum designs D1 and D2 as shown in Table 2. The results for case IV are given in Table 3. For initial designs I1, I2, and I4, optimum design D5 was obtained with structural weight $W = 10.0$, $\omega_2 = 3.0$, and $S_G = 923.57$. This design was different than that obtained for cases I and II. Optimum design D2 was obtained for initial designs I7, I8, and I11. For case V, the problem was solved for a number of initial designs. Most of the time, the same optimum design D2 was obtained except for the initial designs I2 and I9, resulting in design D6 (which has nearly the same weight as design D2). However, the control gain norm S_G for designs D2 and D6 was equal to 1500 and 245.4, respectively, and the areas of the members for the two designs were switched.

Discussion of Results

Computer simulations of the horizontal deflection U_1 and the input $F(t)$ for six different optimal designs D1–D6 are given in Figs. 2 and 3. The infinite time control cost matrix for each design was computed by solving the associated Lyapunov equation and the control effort was evaluated for an initial state $X_0 = [1\ 1\ 0\ 0]^T$. In Table 5, we have listed S_G, J, structural natural frequencies ω_1 and ω_2, closed-loop damping ratios ζ, closed-loop natural frequencies $|\rho_i|$, and the control effort per unit structural weight J/W. Also listed are the ratios of the closed-loop natural frequencies to the open-loop natural frequencies[11] $\lambda_i = |\rho_i|/\omega_i$. We note that, if a sufficiently large sample of initial states were taken, the expected control effort $E(J) = S = S_G$. Even with one initial state tried for illustration, Table 5 corroborates this expectation.

The first four optimal designs listed in Table 5 have an equality constraint of 1500 on the gain norm. From Fig. 3 and Table 5, we note that the higher control efforts of D1 and D4 are due to the higher control input magnitudes. The lower control efforts for D2 and D3 are a result of the lower magnitudes required by these designs. Even though the gain norms are the same for all four optimal designs D1–D4, from Table 1 and the form of the feedback input [Eq. (31)], we conclude that nonuniformity in the signs of the control gains produces smaller control inputs and hence lower control effort. Conversely, uniformity on the signs of control gains produces larger input magnitudes and hence larger control effort. Simulations of designs D1–D4 and Table 5 show that the settling time is reduced not as the control effort increases, but as the control effort per unit weight increases.

In obtaining D5 and D6 of Tables 3 and 4, the purpose was to reduce the control gain norm by putting a lower bound constraint on ω_2, thereby making it closer to the second closed-loop natural frequency $|\rho_2|$. As a result, the control gain norm for designs D5 and D6 was reduced considerably in comparison to designs D1–D4. Furthermore, design D6 had a smaller gain norm and control effort than those of design of D5. Any possible tendency toward larger control inputs because of the uniformity of gain signs is offset by the considerable reduction in the gain magnitudes of design D6.

Both D2 and D6 have the same minimum weight, same structural frequencies, and almost the same control effort. However, D2 is obtained for a larger control gain norm of 1500 vs a gain norm of 245 for D6. Optimization achieves this simply by interchanging the cross-sectional areas of members 1 and 2 between designs D2 and D6. Even though both structures are equivalent mechanically, from a control design point they are drastically different configurations. Note also the considerable difference in the settling time between the two designs. Although D6 has a lower control gain norm, its control input is almost the same as that of D2. In this regard, the effect of uniformity in the signs of gains of D6 is offset by the reduction in the magnitudes of gains when compared to D2.

In all of the optimal designs listed in Table 5, it is observed that the control effort increases as the magnitude $|\lambda_2 - 1|$ increases. A ratio of $\lambda_2 = 1$ means that the controlled natural frequency is the same as that of the uncontrolled natural frequency. Any deviation of λ_2 from unity implies stiffening ($\lambda_2 > 1$) or softening ($\lambda_2 < 1$) of the structure by the control system. A change in the stiffness requires a larger control effort over that of a control design that does not change the stiffness. Hence, as $\lambda_2 \to 1$, lower control efforts are realized.

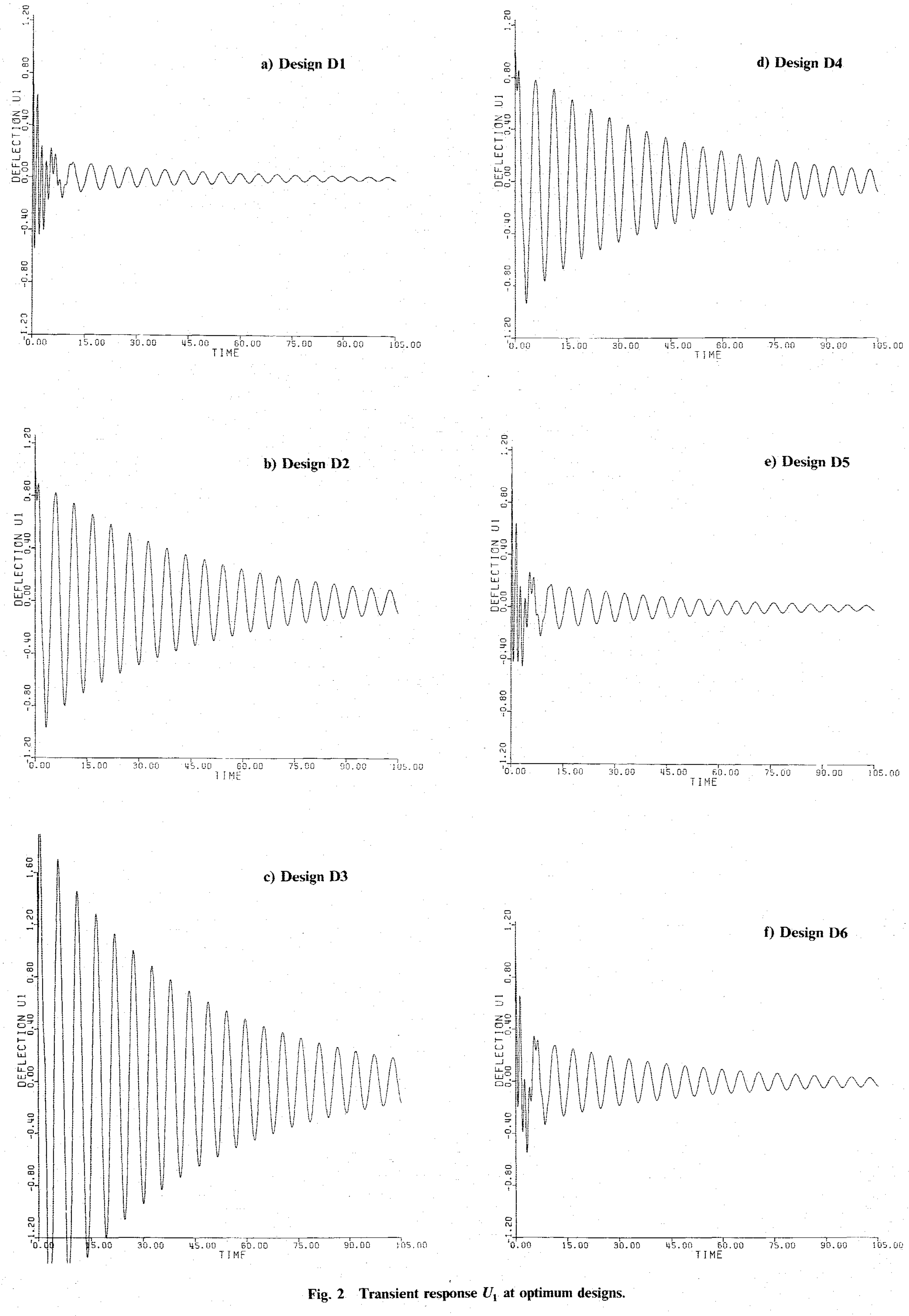

Fig. 2 Transient response U_1 at optimum designs.

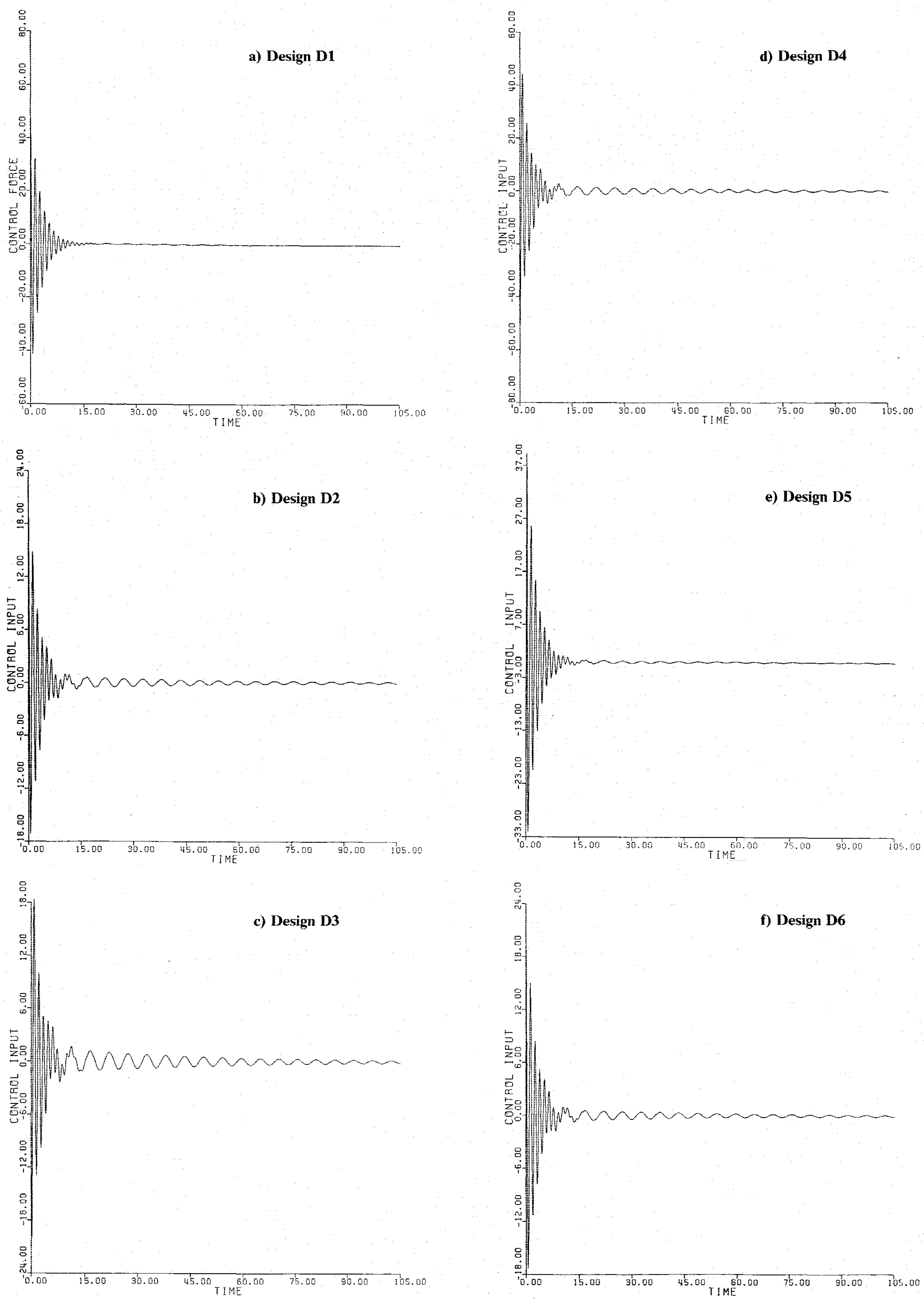

Fig. 3 Control input force at optimum design.

Furthermore, control effort can be shown to be proportional to the cube of the open-loop structural frequencies.[11] The implication is that for a given $|\rho_i|$, λ_i and ω_i will have opposite effects on the control effort. For example, the lowering of ω may force a reduction in the control effort, but since the separation $|\lambda - 1|$ will increase simultaneously, there will also be a tendency to increase the control effort. The net change in the control effort will depend on which effect is more dominant. As a consequence of these trends, a heavier structure does not necessarily require a larger control effort nor does a lighter structure necessarily imply a lower control effort. Table 5 verifies these trends. Larger separations of λ_2 from unity increase the control effort, the only exception being between designs D1 and D4. Design D4 has smaller separation of its λ_2 than that of D1. However, D4 has a higher ω_2 than that of D1. The second effort appears to be more dominant, such that a larger control effort is needed for D4. The implication of these observations is that some frequency constraints and closed-loop eigenvalues constraints may impose frequency separations $|\lambda - 1|$ that cannot be satisfied simultaneously by the prescribed constraint on the minimum gain norm S_G. Thus, a mechanism by which the feasibility of the solutions can be affected is clearly identified in the proposed formulation of structure/control system optimization.

The final choice among the locally optimal designs listed in Table 5 should be a matter of compromise among the control effort, weight, and smallest settling time. D1 has the lowest weight and settling time, but it requires a comparatively high control effort with a gain norm of 1500. With a compromise in the structural weight, D6 still maintains good settling time with 84% reduction in the gain norm and 82% reduction in the control effort over D1. From the performance points of view D6 should be preferred.

Conclusions

A simultaneous structure/control system optimization has been formulated. The formulation demonstrates that optimum structural designs can be obtained while using the Frobenius norm of the control gains as an effective constraint along with structural frequency constraints to monitor the transient response and the control effort for the closed-loop control system. Both qualitative and quantitative observations of the parameters that govern the performance and nature of the optimum structure/control system have been made. The formulation has been applied to a two-bar truss structure with a single input. Application of the formulation to multiple input large-order systems remains to be demonstrated.

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